Wordsworth and non-Euclidean geometry

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This thesis is my own work. All sources used have been acknowledged.

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I'd be lying if I said the last two years had been easy, but I would like to acknowledge that not once did I get bored with my topic. On the contrary, I consider myself fortunate to have been able to pursue two skeins of thought which not only interest me greatly but were the main concerns of my late intellectual hero, Jacob Bronowski: mathematics and poetry.

Contents

Introduction

A little known fact about William Wordsworth is that he had a profound admiration for geometry (Gill 107; Jenkins 2008, 72; Legouis 86; Thompson 92). He regretted not pursuing his mathematical studies beyond his time at Cambridge and felt in later life that poetry and geometry represented "the two hemispheres" that "compose the total world of human power" (de Quincey 169). This level of reverence for geometry is evident in several passages of his poetic output, but most significantly in a section of Book V of *The Prelude* and again in a section of Book VI (see Appendix A for both passages). In the latter, Wordsworth characterises geometry as: "an independent world, created out of pure intelligence" (166–7). Such lines must resonate with anyone familiar with the remarkable story of the development of non-Euclidean geometry. The romantic hero of that nineteenth century exploration was the Hungarian mathematician and polymath János Bolyai, who exclaimed in a letter to his father — in a quotation often repeated in histories of mathematics — "Out of nothing, I have created a strange new world" (O'Leary 394). He was referring to a hypothetical world he had created which followed a geometry unlike that of standard Euclidean geometry, in which even our commonsense notions of parallel lines do not exist.

The affinity between the lines from *The Prelude* and the lines from Bolyai, is more than lexical and indeed the quotation from Bolyai is variously translated as: "I have created a new universe out of nothing", "out of nothing I have created a new and astonishing world" and variants thereof (Eves 60; Faber 161; Gray 1979, 97; Kline 1956, 410). The real affinity relates to the fact that the developments of non-Euclidean geometry instigated a fundamental shift in thought about what mathematics is (Taylor 75–6). Prior to this shift and at the time Wordsworth produced his principal works, geometry, and in fact all branches of mathematics, were seen as being closely linked to the real world and physical phenomena. This view reached its apogee during the high Enlightenment of the second half of the eighteenth century (Kline 1965, 286). Wordsworth, writing against this tradition, with its mechanistic worldview and emphasis on analytic reasoning, was taking up a position that, as we will see, was completely unorthodox in the mathematical community of the time (Alexander 182).

Non-Euclidean geometry (hereafter NEG), along with some other key results in other branches of mathematics in the second half of the nineteenth century, caused mathematicians themselves to veer away from the Enlightenment view of mathematics, closely allied as it was to natural science, towards a more platonic conception of mathematics, which sees mathematical objects as abstract, immaterial, transcendent and independent of the real world (Kline 1980, 322–4). Such a position is now orthodox among mathematicians and is evidence of the separation which still exists between the natural sciences and mathematics. As such we find that intellectual history has actually been kind to Wordsworth's characterisation of geometry and that his views of geometric "abstractions" are mainly in accordance with what most practising mathematicians believe today (Gray 2008, 441).

The great majority of scholars of English literature have overlooked Wordsworth's interest in geometry and none have looked at the significance of later developments in geometry with respect to his views on the subject. That such an esoteric mathematical topic rarely captures the interest of those studying poetry no doubt partly explains why the connection seems not to have been made before. We might also conjecture that few mathematicians have shown much interest in the autobiographical poetry of William Wordsworth and that such investigations may seem as abstruse to them as NEG appears to students of English literature. In any case, neither discipline has engaged substantively with this connection and therefore this thesis takes an interdisciplinary approach, seeking to examine literary history and the history of mathematics in conjunction. The relevant passages of text will be analysed to elucidate Wordsworth's views and to show how they relate to modern ideas in mathematics, but the bulk of this thesis does not comprise a close textual analysis. Because the salient point is the affinity between Wordsworth's writing on geometry and later mathematical orthodoxy and because this has not been described previously, the scope of this investigation is to examine the existing but insufficient literature on Wordsworth and geometry and to demonstrate the significance of this affinity in terms of intellectual history.

Thus, Chapter 1 opens with a brief review of Wordsworth's views on geometry as evidenced in his poetry; it then examines the modest amount of literature on Wordsworth's interest in geometry and identifies gaps in that literature, where a knowledge of later developments in mathematics and geometry would have been useful. Chapter 2 briefly covers the development of NEG and its implications for pre-existing ideas about space, the universe and knowledge, demonstrating the broader historical significance of the topic. Chapter 3 locates Wordsworth's views on the spectrum of opinions currently held by mathematicians and philosophers of mathematics; it then returns to the poems with which we started, to examine their representation of geometry, recast in the light of the intervening history of geometry and modern mathematical ideas; the chapter closes by offering some suggestions for further research.

Chapter 1: Wordsworth and geometry

This chapter contains a review of the literature on Wordsworth and geometry. The first section outlines the textual evidence of the poet's interest in the subject, primarily as evinced in *The Prelude*. The second section then reviews the scholarly literature relating to Wordsworth and geometry and proceeds conceptually from the biographical evidence for his interest in the subject, to the influence of Newton, to the distinction between natural science and mathematics in Wordsworth's views and ends with the three main philosophical concepts that characterise his views on geometry: abstraction, synthesis and transcendence. The third section briefly outlines some of the crucial gaps in the literature and looks at how the scholars who *did* engage with Wordsworth's ideas about geometry, nonetheless overlooked key developments in mathematics at the time and since.

A thorough review of all easily available literature on Wordsworth was conducted (see Works Consulted), with only a handful of sources returning germane results, often simply a single mention of Wordsworth's interest in Euclid, mathematics, geometry, or Newton, *e.g.* Beer (180), Moorman (57), Onorato (371) and Sheats (4) respectively. The result is an exiguous bibliography of works that engage substantively with the topic. Some scholars have noted, in biographical studies, the importance of Wordsworth's interest in the subject (Burton, Curran, Kelley, Lindenberger, Rader) and a few have engaged with the geometric ideas in his poetry (Baum, Durrant, Johnson, Schneider). Surprisingly, the handful of critics who *did* go so far as to focus on the importance of geometry — even indeed to focus on the key passages from *The Prelude* — all neglected to examine how the extraordinary developments of NEG and modern physics actually favour Wordsworth's characterisation of geometry as being abstract, synthetic and transcendent.

With such outcomes, from what is quite a large sample of the entire English speaking scholarly literature on Wordsworth, it can be confidently asserted that few if any scholars have ever engaged with the topic of non-Euclidean geometry with respect to Wordsworth's ideas or his poetry. So although there is a vast corpus dealing with Wordsworth's philosophical views, or tracing the sources of his poetic output, the literature concerned with his interest in geometry can be summarised fairly exhaustively.

Geometry in Wordsworth's poetry

The primary evidence for the character of Wordsworth's admiration of geometry comes from two key passages in *The Prelude* (see Appendix A). First is a section of almost a hundred lines in Book II (49–140; all references to *The Prelude* are for the 1850 edition unless otherwise indicated), which describes an allegorical dream sequence, wherein the speaker of the poem, while reading *Don Quixote* and ruminating on "poetry and geometric truth" drifts off to sleep. He dreams of an encounter with a Bedouin tribesman bearing two sacred objects: a stone and a shell, which he explains are actually books, the stone being Euclid's *Elements*, the shell a book of poetry. Euclid's work is described by the speaker as:

> The one that held acquaintance with the stars,
> And wedded soul to soul in purest bond
> Of reason, undisturbed by space or time; (V 103–5)

This sequence has frequently been read as demonstrating the high esteem in which Wordsworth held geometry — as almost the equal of poetry — and also the transcendent, immutable, eternal nature of it that attracted him (Bewell 250). The vision dissolves when the speaker awakes from his dream, as the shell's poetry — really an apocalyptic ode — presages a calamitous flood. The notion of geometry as being undisturbed by space or time is thereby contrasted with the finitude of the world around us represented by the eschatological imagery.

More detailed is the description of geometry given in Book VI (115–67) which opens with a paean to the transcendent aspects of geometry, along with regret at not studying it more thoroughly:

Yet may we not entirely overlook
The pleasure gathered from the rudiments
Of geometric science. Though advanced
In these inquiries, with regret I speak,
No farther than the threshold, there I found
Both elevation and composed delight:
With Indian awe and wonder, ignorance pleased
With its own struggles, did I meditate
On the relation those abstractions bear
To Nature's laws, and by what process led,
Those immaterial agents bowed their heads
Duly to serve the mind of earth-born man;
From star to star, from kindred sphere to sphere,
From system on to system without end. (VI 115–128)

The speaker continues his praise of geometry by reporting the pleasure drawn from the contemplation of these "abstractions" and how such contemplation was a way to access "the one / Supreme Existence, the surpassing life" which "is, / And hath the name of God". Geometric figures here seem to occupy a special place beyond "Nature's laws" but still of use to the terrestrial interests of humanity, or indeed people on other worlds. There follows an image borrowed from a work by the evangelist, John Newton, of a castaway on an island seeking spiritual respite by drawing the figures from Euclid's *Elements* with a staff on the sand (Jenkins 58). Here, geometry is held up with poetry as the purest expression of imagination and that which is least encumbered by human concerns. Thus we see the shipwrecked figure of Book VI drawing geometric figures in the sand, able to commune still with the eternal and abstract nature of geometry, despite being destitute of all material comforts. This is compared to the sense of order and immanence a poet can obtain from the study of geometry:

Mighty is the charm
Of those abstractions to a mind beset
With images, and haunted by herself,
And specially delightful unto me
Was that clear synthesis built up aloft
So gracefully; even then when it appeared
Not more than a mere plaything, or a toy
To sense embodied: not the thing it is
In verity, an independent world,
Created out of pure intelligence. (158–167)

Here we see that geometry is praised partly because it is a synthesis, something built up into an organic wholeness rather than broken down and dissected into constituent parts. We also see the crucial contrast between geometry as something which is *used* — a "plaything, or a toy" — and geometry as something transcendent and beyond the purely instrumental. Mathematics more generally is praised again for its transcendent and abstract nature in Book XI:

> such sloth I could not brook,
> (Too well I loved, in that my spring of life,
> Pains-taking thoughts, and truth, their dear reward)
> But turned to abstract science, and there sought
> Work for the reasoning faculty enthroned
> Where the disturbances of space and time—
> Whether in matter's various properties
> Inherent, or from human will and power
> Derived—find no admission. (321–32)

Notably the 1805 version read "mathematics" instead of "abstract science" reinforcing the fact that it is the abstractness of mathematics and geometry that delights Wordsworth. Here again is the repeated theme of these abstractions being beyond space and time, independent of the material world. There is also a hint of this reverence for "abstract science" and the consolations it provides in Wordsworth's other long, quasi-autobiographical poem, *The Excursion*:

Lore of different kind
The annual savings of a toilsome life,
His Schoolmaster supplied; books that explain
The purer elements of truth involved
In lines and numbers, and, by charm severe,
(Especially perceived where nature droops
And feeling is suppressed) preserve the mind
Busy in solitude and poverty. (I 250–7)

Together, this and the above passages have been seen by critics attuned to Wordsworth's geometric interests as evidence of his long-held love for the subject and his regret at not pursuing it further (Baum; Durrant; Johnson; Schneider).

Additionally, there are a few other hints scattered through his verse which point to his admiration for and contemplation of geometry. The later poem, "On the Power of Sound", includes several stanzas which glorify pythagorean themes of harmony, both musical and mathematical: "By one pervading spirit / Of tones and numbers all things are controlled" (177–8). *The Prelude*, too, contains an isolated mention of Archimedes, the geometer from Syracuse, as that "pure abstracted soul" (XI 435). Overall there is not much evidence of his admiration for geometry in his wider poetic output and, unlike some of the metaphysical poets like Herbert, Marvell and Vaughan, Wordsworth does not employ geometry for use in poetic figures. But in his autobiographical poetry we find evidence for a highly developed sense of the abstract and timeless nature of geometry, which obviously formed an important part of the poet's philosophical ideas.

A review of the literature

Biographical evidence of his interest in geometry—textual evidence—Newton's influence—geometry distinct from the natural sciences—geometry is abstract—geometry is synthetic—geometry is transcendent

The origins of Wordsworth's interest in geometry are well documented and began when he was at Hawkshead Grammar School in Cumbria (Baum 392n, 394; Johnson 5; Schneider 15, 95–6). He excelled at geometry and had a particularly inspiring teacher, William Taylor, who was himself senior wrangler at Cambridge in 1778 — the second highest mathematics prize — and who encouraged the young Wordsworth's interest in Euclid in particular (Baum 397–8; Schneider 5). Wordsworth then went up to Cambridge and his intellectual explorations during that time provide crucial evidence in explaining what his views on geometry were and how these views fit with the context of his broader worldview. His documented reading from the time period is fairly scanty and he was not a very diligent student, becoming disaffected by the class hierarchy and the competitive, mathematics-based examination system (Schneider 40, 95–8). It is well established that the curriculum for undergraduates at Cambridge was less mathematically involved than Wordsworth's final years at Hawkshead and he arrived with at least a year more reading in mathematics than most of his coevals (Baum 394; Schneider 95). But whereas he had already read the first six books of Euclid's *Elements*, the post-Newtonian emphasis on mechanics (then the study of Newtonian physics, especially concerning applications of differential calculus to practical problems) left Wordsworth uninspired and behind in his studies; the abstract nature of geometry was already far more attractive to the young poet than were the worldly applications of late Enlightenment mathematics (Baum 399; Reed 174). Wordsworth would later confide in his friend, the Irish mathematician, William Rowan Hamilton, that he wished he had pursued mathematical studies further and taken them more seriously while at Cambridge (Baum 392–3; Beatty 233). Thomas de Quincey also records in his essays on the Lake poets that when he visited Wordsworth, the poet proclaimed his love of the higher mathematics, but lamented his lack of diligence in studying it at Cambridge (Bruhn 57).

More ample and better acknowledged by scholars is the evidence of Wordsworth's love of Newton (Moorman 57; Rader 44–6; Thomas 37–9). He had a copy of *The Principia* on his shelf (Shaver 345) and left several laudations in *The Prelude*, writing of "the great Newton's own ethereal self" (III 266) and also putting Newton alongside Shakespeare in greatness (VII 165); but most notable is the following passage:

> And from my pillow, looking forth by light
> Of moon or favouring stars, I could behold
> The antechapel where the statue stood
> Of Newton with his prism and silent face,
> The marble index of a mind for ever
> Voyaging through strange seas of Thought, alone. (III 58–63)

Geoffrey Durrant is perhaps the scholar most attuned to Wordsworth's interest in geometry and, in his *William Wordsworth*, he reads this passage as being illustrative of Wordsworth's reverence for how Newton's achievement was chiefly a product of quiet intellect, pure thought and not an engagement with the busy world (131) as contrasted with the description of Trinity College life which precedes it (III 48–52). It can also be easily compared with the key passage from Book VI, wherein geometry is offered as a refuge to poets whose minds are “with images beset”; that Wordsworth saw in his own poetic endeavours a parallel with Newton’s mathematical achievements is well established (Durrant 1969, 132–3), although as Melvin Rader observes, it may seem to run counter to one’s expectations of Wordsworth:

> At first glance it may appear unlikely that he could have been much influenced by Newton, since the connotations of the Newtonian theory seem uncongenial to romantic poetry. According to the prevalent interpretation Newton had furnished scientific proof that the universe is a perpetual motion-machine constructed by a kind of Divine Mechanic. (Rader 44)

And yet Newtonian science, although it was the apotheosis of the mechanistic philosophy of the Enlightenment, could also be seen as an extension of Euclidean geometry into a more generalised, abstracted form, as will be discussed in Chapter 2. It needs to be kept in mind that Newton, although he privately worked out most of the calculations for his *Principia* using his own device, calculus (he called it “fluxions”), wrote up the work using Euclidean derivations for all his statements about the physical laws of the universe (Newton 2013, 95–102, 261–3, *etc*.). Newton himself, writing anonymously, explained the choice:

> By the help of the new Analysis [calculus] Mr. Newton found out most of the Propositions in his Principia Philosophi: but because the Ancients for making things certain admitted nothing into Geometry before it was demonstrated synthetically, he demonstrated the Propositions synthetically, that the Systeme of the Heavens might be founded upon good Geometry. (Newton 2002, 20)

Note that Newton distinguishes between the synthetic study of geometry, which for him involved building from a few axioms and figures, a whole geometric description of a complicated system, and the analytic study of calculus (still referred to as analysis in mathematics) which would take the system and then break it down into limits, infinitesimals or derivatives. As such Newton presented the ultimate use of Euclid's synthetic style, applying those abstractions to the underlying laws of nature and hence was seen by Wordsworth as describing nothing less than "the mind of God" (Durrant 1970, 8; Johnson 50). Jones even contends that Wordsworth's whole poetic enterprise can be seen as an attempt — a failed one — to extend via Euclid, thence Newton, thence Wordsworth, a line of continued abstraction in Western thought (40); but this is a somewhat overextended reading. In fact, despite Wordsworth's struggle with the Newton-focused curriculum at Cambridge, it did serve to consolidate his love of Newton and vicariously of geometry (Rader 95–6). So whereas Wordsworth hated the practical problems of calculus to which he was subjected, he was quite impressed by the other primary application of Newtonian physics, namely that of astronomy. Durrant, in *Wordsworth and the Great System* associates Wordsworth's interest in Euclidean geometry with his interest in and reverence of Newton and with the employment of Euclidean geometry in Newton's great system of the universe (21–4). After quoting lines 115–28 of the 1850 version of *The Prelude*, Durrant writes:

> This passage makes plain that 'geometric science' was, for Wordsworth, intimately linked with Newtonian astronomy and the natural laws — with that region of 'science' for which Wordsworth so often expresses his deep admiration, and with which he records none of the dissatisfaction aroused in his mind by the biological, psychological, and social sciences as practised in his day. It is in this area of science that Wordsworth is most deeply interested; and there we shall find the strongest influence on the actual texture and form of his poetry.

(Durrant 1970, 21)

Indeed Durrant and others (Baum, Johnson, Rader, Schneider) do well to separate the assumed anti-science stance of Wordsworth from his love of the abstract nature of Newtonian and Euclidean ideas. *Abstract* is the key word and is used to describe geometric figures once in the 1805 text and twice in the 1850 version. Durrant presses the point that whereas the chemical and biological advances being made at the time jarred Wordsworth's Romantic sensibilities, he saw in Newton's great synthesis of mind and matter something of the transcendental, which he coveted above all else, especially in his early career (Durrant 1970, 2–8) — perhaps even the famous motion and spirit that "rolls through all things" from "Tintern Abbey" (Rountree 25). Schneider also links his love of Newton's abstractions to the central philosophical concerns of Wordsworth's transcendentalism:

> Moreover, the relation of geometry to those phenomena was abundantly clear to any reader of Newton's *Principia*, for that book was no more than geometry set in motion, the motion of the sun, planets, and comets. The geometrically expressed laws of nature were another form of the 'brooding presence'. (Schneider 257, quoting Whitehead 117)

The “brooding presence” is A N Whitehead’s phrase from his highly influential *Science and the Modern World*. As a few commentators have pointed out, Wordsworth’s love of Newton is not in contradiction with his broader distaste for science. Whitehead did much to rehabilitate Wordsworth’s reputation from that of someone who hated all science to one who revered the kind of science represented by Newton, but who disliked the science of experimentation (78–80; Schneider 253). Whitehead’s discussion of Wordsworth’s views of nature is actually most concisely expressed by Schneider, who notes that Whitehead recognised that Wordsworth’s love of a certain *kind* of science is linked with his love of Newton, as opposed to, say, the chemistry of the day, because that kind of science “murders to dissect”, as in the phrase borrowed from “The Tables Turned” in *Lyrical Ballads* (Schneider 255–6). Again it is the abstract nature of Newtonian science which attracted Wordsworth and as Whitehead points out physics has only become more abstract since Wordsworth’s time (Whitehead 133–5). Along with abstract qualities, the two other features of geometry which make it attractive to Wordsworth and which allow it to sit with his philosophical tendencies are its *synthetic* nature and its *transcendental* nature.

Dealing first with the abstract nature of geometry, the *OED* has several contemporaneous senses of the word, the two most likely being two of the most common today: abstract, meaning something that is removed from physical reality or the opposite of concrete; and abstract, meaning something which has been generalised to its common or important features, its details stripped away. Both senses fit with mathematical ideas about what geometric figures are: either totally immaterial objects which cannot exist in physical reality, or figures which retain the most important characteristic but none of the details of objects in the real world (*i.e*. the three sidedness of a perfect geometric triangle and its rougher correlates in the real world). In all, Wordsworth uses the word abstract or variations of it, ten times in the 1850 text, in four different senses, with five of the instances being to do with abstract as the opposite of concrete. Abstractness is the main attribute of geometry that Wordsworth praises and Curran argues that it fits within a wider project of Wordsworth's (118), related to articulating the proper content of poetry, that it be neither mannered imitation of older forms or "specious and sensational novelty" (132).

Another key attribute is the synthetic nature of geometry. Wordsworth wrote:

> Mighty is the charm
> Of those abstractions to a mind beset
> With images, and haunted by herself,

And specially delightful unto me
Was that clear synthesis built up aloft
So gracefully; (VI 158–63)

Of particular importance is the use of the words "synthesis" and "synthetic" as antonyms of analysis and analytic. Again, the *OED* gives several possible senses. The most established sense in the English language in 1805 was simply that analysis involves breaking into parts as opposed to synthesis which involves combining into a whole; but there is also the Kantian sense, which the *OED* notes had entered the language late in the eighteenth century and which pertains to synthetic knowledge, *i.e.* where a proposition's truth is not evident in its subject's predicate. Euclidean geometry was Kant's famous and only example of synthetic *a priori* knowledge. This synthetic–analytic dichotomy is noticed by Johnson who, following on from his quotation of the same passage of Durrant (1970, 21) referred to above, argues:

> That is to say, astronomy and mathematics exercise the synthetic powers of reason which are synonymous with the rational imagination, but the life-sciences rely on merely analytic reasoning which breaks down and classifies things as particulars and loses the sense of interconnections among them. One might even add that Wordsworth's

> pleasure in geometry is partly attributable to his desire to employ its forms in the description of nature and the passions and thus to parallel Newton's achievement in using Euclidean proportions for an account of planetary and stellar motion. (Johnson 81)

Again we see the connection made with Newton and again the distinction is with the other emerging natural sciences that "murder to dissect", where geometry is based on imagination and drawing together: *i.e.* synthesising. As for the Kantian sense of the word, Rader makes the case that this is precisely the kind of point that Coleridge communicated to Wordsworth and that would have bolstered Wordsworth's ideas about the imagination being supreme (182–3). In support of this, Rader refers to a passage from *The Prelude*:

> Science appears but what in truth she is,
> Not as our glory and our absolute boast,
> But as a succedaneum, and a prop
> To our infirmity. No officious slave
> Art thou of that false secondary power
> By which we multiply distinctions; then,
> Deem that our puny boundaries are things
> That we perceive, and not that we have made. (II 212–9)

For Wordsworth the primary power is “the synthesising process, which takes place before experience and renders experience possible”, with the analytical, experimental or empirical distinctions of the new sciences being secondary (Rader 183).

The final point about the nature of geometry that makes Wordsworth aptly placed to hold a view commensurate with later mathematical orthodoxy concerns whether geometric or mathematical objects are independent of human thought. Here we can see there is a blurring of distinctions, partly because of Wordsworth’s inconsistent and not completely elaborated philosophical system (Curran 118–9; Hirsch 1960, 29; Schneider 241), especially between what would normally be called idealism and transcendentalism. Idealism — defined most simply as a belief that the world is created by the mind — as Rader notes, is allied to transcendentalism, which latter is normally taken to mean that the mind can transcend experience in order to gain knowledge (Rader 30–1). These two notions can obviously go together and frequently do and the issue is further complicated by Kant’s insights and the development of his own form of idealism named “transcendental idealism”. But there is a crucial difference concerning whether the world, or some aspect of the world including an immaterial one, traditionally God, can be independent of human thought. This is where true idealism would regard objects, including mathematical objects, to be inseparable from their apprehension by the human mind (Scruton 100–1). But although Johnson seems at first to run these ideas together (38–9), the following passage outlines the basis of Wordsworth's transcendentalism, which is not idealism:

> Like the Pythagoreans and Platonists before him, Wordsworth reasons that geometry does not depend for its existence on the life of the mutable universe. Take away the whole of external and human nature, and geometrical relationships would continue unabated. This is, for Wordsworth, a 'paramount belief' and one of his profoundest convictions. In geometry, he recognises 'a type, for finite natures, of the one / Supreme Existence,' in which mathematical thought is sustained; and although such thought does not require the existence of the human mind, the ability of the mind to conceptualise an abstract, semipiternal [sic] emblem of the Godhead is evidence that the nature of the mind transcends temporal limitations. (Johnson 81–2)

The transcendental aspect of geometry, which Rader (30) takes to be the most important aspect of Wordsworth's early philosophical system, is certainly shared by later unintuitive NEGs. It is this independence from human thought which ultimately characterises Wordsworth's views of mathematics as what we would now classify as a form of mathematical platonism. His friend Hamilton recorded a discussion the two had regarding the "abstract forms of mathematics" and "whether they were a link between the human and the divine" (Beatty 233). Wordsworth reportedly smiled and said he was:

> reminded of the Platonic doctrine of the internal existence in the marble of those beautiful forms from which the sculptor was supposed only to withdraw the veil. (Hamilton in Beatty 233)

Such sentiments as these at least serve to link Wordsworth's generally overlooked interest in geometry with his larger and better documented interest in philosophical ideas, which were chiefly influenced by readings of Plato, David Hartley, William Godwin, William Paley and his friendship with Coleridge through which he learned about Kant (Rader 35). Of these, he would eventually reject Hartley, Godwin and Paley (Schneider 241) and the influence of Coleridge served mainly to inculcate a certain admiration of Kant, although he probably had not actually read anything by Kant himself (Rader 66).

Durrant asserts that Wordsworth owes as much to Kant as to Euclid and Newton, who together form a triumvirate of thinkers who mapped out the world and its form in time and space (Durrant 1970, 94–6). Euclid laid down the rules of geometry, apparently so logical and seemingly confirmed every time an architect designs a building; Newton's idea of absolute space seemed to apply a grid-like structure of rectilinear orderliness to the entire cosmos, arguing that the laws of motion and properties of space are uniform throughout the universe; and Kant incorporated this view of space into his own great system of metaphysics, reasoning that the fundamental rules of space and time must be an innate part of the subject's perception of the world. As we will see in Chapter 2, certain of the propositions of Euclid no longer hold — some are denied by experience (synthetic *a posteriori* knowledge), others by mathematical investigation (synthetic *a priori* knowledge) into NEGs (Jenkins 59; Shapiro 79–81). Certain of Newton's ideas have also been disconfirmed by subsequent developments in physics, most notably by Einstein who utilised NEG and in so doing demonstrated that the laws of motion are relative rather than absolute and that the geometry of space is locally altered by gravity (Grayling). These developments called into question Kant's notion of the knowledge of space and time being verifiable because it is built in to our understanding of the world; it transpired that our intuitions about the nature of space and time are naive and that our understanding of mathematics at the time when Kant was writing was very narrow (Eves 68–9).

Overall, Wordsworth's philosophical allegiances are complicated and it is easier to draw out some concepts to which he was devoted than to attempt to imbricate his various ideas onto the body of work of a particular thinker. Abstraction, synthesis and transcendence are all concepts he was enamoured of, but they do not form a philosophical system, although these three do all appear in platonic theories of mathematics; as we will see in Chapter 3, however, they are the key tenets of mathematical platonism, rather than platonism qua the philosophical system of Plato. In fact, most critics writing incidentally of Wordsworth's own philosophical system contend that he simply did not have one (Curran 118–9; Hirsch 1960, 29; Schneider 241; Sheats 4). The authors who attribute to Wordsworth a more elaborated, structured and consistent philosophy tend to be those who are writing specifically on Wordsworth's philosophical or metaphysical ideas (Durrant 1970; Johnson; Rader); this might be because they have interrogated the ideas with greater attention or merely because of a selection bias.

Gaps in the literature

Conflation of maths and science—ignoring the Enlightenment dogma of maths—significance of abstract nature of geometry—NEG, Einstein & later physics

The writers who *have* examined the significance of geometry in Wordsworth's thought and in his poetry, generally exhibit similar limitations. A conflation of mathematics with the natural sciences is an understandable but crucial error found in some commentators' work. Other critics have sought to repudiate the claim that Wordsworth is anti-scientific and in so doing have pointed to his love of geometry as expressed in *The Prelude* (Holmes, Kelley, Shaw) not recognising that the features of geometry which make it attractive to Wordsworth are those which are missing from the natural sciences.

There is also a widespread lack of recognition of the Enlightenment orthodoxy in mathematics, compared to which Wordsworth's views are unusual for the time. Of course the received history of the Romantic epoch is that there was an intellectual revolt against the mechanistic, mercantile, empiricist worldview of the eighteenth century (Hogle 5; McGann 23–7) and so Wordsworth was standardly painted as some kind of nature-loving, anti-Enlightenment figure who was opposed to the intellectual milieu in which he was raised. This is a caricature, but contains some truth; interestingly, few of the critics who do recognise the importance of geometry to Wordsworth realise that the empiricist–transcendentalist tension is particularly significant in mathematics, because unlike in other fields of inquiry, the transcendentalists ended up in the ascendency. However, the Foucauldian view of the history of thought, for example, holds that reason, aided by analysis and scientific discourse, became the dominant discourse in the late sixteenth century and continued until the nineteenth century when it began to be challenged and undermined (Foucault xxi–xxiv). The thought of the time during *The Prelude's* composition was definitely still hostile to platonism, abstraction and synthesis, influenced in mathematics by Descartes' analytic method (Foucault 52–8). But almost a century before a similar attack from the so-called human sciences, pure mathematics, led by geometry, fought and won several battles against the Aristotelian, cartesian, Enlightenment *episteme* dominant in European thought (Alexander 182; Shapiro 73–7).

Because of an ignorance of this development and the aforementioned conflation of mathematics and science, even the critics who have engaged with Wordsworth's interest in geometry *and* correctly recognised it as a love of abstraction, synthesis and transcendence — have failed to recognise the significance of such views in light of developments in NEG, let alone its implications for Einstein's discoveries, let alone modern physics or mathematics. The few critics who at least made it that far will briefly be examined to see how they neglected the history of mathematics.

Schneider places great emphasis on geometry and particularly its expression in Newton and its abstract nature. He also comes close, in a passage already referenced here several times, to engaging with the abstract nature of twentieth century physics, via a discussion of Whitehead's work (Schneider 248–56). But although he discusses geometry and mentions the new abstract nature of physics, he fails to make the connection between the two; *viz.* that

Einstein used NEG and that NEG was the main development which instigated the general movement towards abstraction in mathematics and physics (Kline 1956, 466).

Durrant focuses on Newton and also recognises that abstraction is the key to Wordsworth's love of him and Euclid. But he does not go at all beyond Newtonian science and mathematics, despite having Schneider's work to draw on, which he does as early as the preface (Durrant, 1970, viii).

Johnson provides easily the most elaborate study of geometry in Wordsworth's poetry and naturally includes a sizeable section on *The Prelude*. The book argues that geometric proportions are a crucially overlooked aspect of Wordsworth's poetic output. Johnson claims that not only are many of Wordsworth's poems divided into sections reflecting classical proportions, but that the poet's knowledge of geometry is evident at all scales, down to the balance of syllables in an individual line of verse (Johnson 30). Johnson's analysis is thorough and, if we accept his central thesis, then it has large implications for the study of Wordsworth. But Johnson is also typical of the problems of conflating mathematics with science and also of ignoring the post-Enlightenment fate of mathematics. He suggests that Wordsworth's predilection for the transcendent status of geometry has been seen as bizarre at all occasions since his time (49). Johnson claims that since Wordsworth's lifetime, people have "neglected the epistemological aspects of mathematical thought by taking it for granted merely as a tool" (50). And yet it is in fact the period of time since Wordsworth's death in which a great shift occurred in mathematical thought. Whereas the physical sciences were, of course, preoccupied with using mathematics as a tool, pure mathematicians were, at the beginning of the nineteenth century, undergoing an almost neo-platonic, not to say Romantic, transformation of the epistemological basis of mathematics. Further, there is the deliberate emphasis by Wordsworth in the key passage on geometry being not just a "plaything or a toy", but something transcendent. Surprisingly, then, for a work which chides English professors for neglecting the geometric mode of analysis, the work includes only one, dismissive, mention of NEG and deals barely at all with any developments in geometry since the days of Euclid:

> Wordsworth's attitudes towards geometry therefore look towards the past rather than anticipate the future, for they are not particularly compatible with those promulgated since the time of Kant. Geometry has left its Euclidean certainties, and modern philosophy has dealt rather harshly with metaphysics in general, although one of its most frequent debates concerns the possibility of innate or synthetic *a priori* knowledge, which Kant based partly on the existence of mathematics and which expresses the necessary conditions or laws governing the

perceptions of all objects of all thought. (Johnson 48)

Apart from this note on the story of Kant's use of geometry and the subsequent decentring of previous mathematical certainties, Johnson avoids investigating the source of or philosophical implications of Wordsworth's love of geometry. This means that he misses the fact that Wordsworth's attitudes are, somewhat remarkably, very compatible with the ideas promulgated since Kant, right up to the present day.

The most recent commentator on Wordsworth and geometry is Joan Baum, who benefits from Johnson, Durrant, Schneider, *et alia* and whose article, "On the Importance of Mathematics to Wordsworth", currently stands as the summative work in the literature. Baum understands the differences between mathematics and the natural sciences and sets out a delineation early on, drawing on Whitehead and Durrant. Yet even Baum, whose analysis is otherwise exemplary, seems to have critically misread the history of mathematics. Tellingly, Baum does not cite any sources to back up her contentions about the history and philosophy of mathematics, the general trends of which she seems to take as given. In one of the concluding paragraphs of the article, Baum wants to rescue Wordsworth from a perceived moving-on of history:

> Mathematics is what endures "undisturbed," or as the Wanderer says in Book 4 of *The Excursion,* "the measures and the forms, / Which an abstract intelligence supplies" are reflections of a kingdom "where time and space are not" (74–76). Such sentiments may be unsound from the point of view of contemporary mathematicians, but they reflected Wordsworth's views, Hamilton's philosophical principles, and the general attitude toward mathematics in the early nineteenth century, before more advanced mathematical ideas from the Continent took hold at British universities. (Baum 405)

More advanced ideas certainly did arrive from the Continent, in the form of NEG and other developments like abstract algebra and formal logic (Resnick). Together they rendered the "abstract science" that Wordsworth lionised, all the more appreciable compared to the mechanistic, Aristotelian ideas that became unsound and which remain so to this day. Baum, who is attuned to the traditional assumption that Wordsworth and other Romantics disliked science and who distinguishes between mathematics and the natural sciences, is caught out by another assumption about Romanticism, that it does not live on in later mathematics:

> One healthy mathematical change that did begin to take place in the

> early nineteenth century, however, did not enlist Wordsworth's sympathies. In 1850, when *The Prelude* was published, the ideal world of immutable concepts Wordsworth had celebrated as "geometric science" had been fragmented irreparably. The center did not hold: geometry was divorced from algebra, theory from applications and engineering. Time and space had become a calculus of finite differences, and the empirical and analytic were replacing the synthetic and classical as modes of scientific discourse. (Baum 396)

Again, there are no references for any of these claims, which are something of a grab-bag of dubious statements about the history of mathematics. Geometry was in fact reunited with algebra in the mid nineteenth century, as the insights of NEG and abstract algebra led to the development of algebraic geometry (Boyer 520). Theory *was* divorced from applications and engineering but the analytic had replaced the synthetic in the seventeenth century (Foucault 57) and space and time had been discretised by calculus before Wordsworth was born. Later in the same passage, Baum becomes the only scholar found who actually mentioned NEG in a discussion about Wordsworth's views of geometry and mathematics:

> Belatedly, but for its own reputation, Cambridge was beginning to

respond to Continental emphases on the differential and integral calculus and non-Euclidean geometry. (Baum 397)

Unfortunately, this too is not footnoted and is demonstrably false. Schneider documents at length the presence of, at least differential, calculus at the Newton-obsessed Cambridge of Wordsworth's youth. As for NEG, we have seen that it did not even become widely accepted on the Continent until the mid 1850s, after Wordsworth's death. Attempts were made to contact the author, who is retired but still alive, which, if successful, may have consolidated or complicated this critique of the article. At the time of publication neither of these attempts had been successful.

It might seem an audacious proposition that Wordsworth's extolling of geometry in *The Prelude* is not just a matter of him advancing a love for Newtonian certainty (Onorato 371–2), or an aesthetic affinity with geometric proportion (Johnson), or a reaction to Godwinism (Durrant 1970) — but in fact a perceptive mind arriving at a modern conception of geometry, which the mathematical community would not arrive at until the development of NEG and other advances in higher mathematics (Alexander 181–2). But it is in fact based on as much textual and biographical evidence as the readings of Durrant, Johnson, Baum, *et al*. which is to say, not a particularly large amount. Those scholars rely on knowledge of Wordsworth's schooling, the Cambridge curriculum, his known interest in certain philosophers, the influence of Coleridge and the intellectual milieu in which Wordsworth and other poets were participating. Using the same sources but adding a knowledge of subsequent developments in the history of mathematics this thesis aims to show that an entirely valid reading of *The Prelude* is one which interprets Wordsworth as displaying an appreciation for the abstract nature of geometry which was ahead of his time and which resonates today. The following chapters review those intervening developments in geometry and then compare Wordsworth's views to current positions commonly held among mathematicians and philosophers of mathematics.

Chapter 2: non-Euclidean geometry

Geometry is a branch of mathematics, normally defined as the study of points, lines, surfaces and solids (Gulberg 386). Although it is a very familiar topic to everyone with a high school education, the most interesting developments in geometry, developments which had a massive impact on mathematics more broadly, concern ideas well beyond high school level mathematics. The geometry studied in secondary school is all Euclidean; that is, it concerns concepts and theorems, all of which are derived from the work of Euclid (c.300BC). Only in university mathematics and physics courses will students encounter NEG, a highly unintuitive subject but one which has been crucial in the last century and a half of developments in mathematics, physics and philosophy (Detlefson; Eves 67–9; Jenkins 2007, 160–1).

Within the history of mathematics the advent of NEG is one of the signal events and most historians of mathematics hold that it is *the* event which brought about the profound nineteenth century shift in thinking about what mathematics is (Bell 330; Boyer 586; Eves 67; Kline 1956, 431, 466). The relationship between mathematical objects and physical reality is the central question of the philosophy of mathematics and an important question in epistemology more generally. The Enlightenment idea of this relationship was simply that mathematics described reality and was a tool to be used to model it (Alexander 182). Following developments in geometry and some other branches of mathematics, the idea emerged that mathematics could also describe things that do not exist, or that are beyond physical reality: abstract objects, the nature of which so attracted Wordsworth to geometry.

This chapter first gives a brief history of geometry from Euclid to modern physics, paying special attention to the advent of NEG. This shift is also of great importance to modern thought more broadly and the second part of the chapter looks at how NEG influenced mathematics, physics and philosophy. In looking at this history of geometry, it becomes evident that although Wordsworth's characterisation of geometry was eccentric or even misguided by the standards of his time, it fits more neatly with the views which would become orthodox a generation later and which predominate in the mathematical community to this day.

A brief history of developments in geometry

Euclid—geometry as a tool—non-Euclidean geometry—pure intelligence

Euclid's *Elements* (first published c.300BC) remained virtually unchallenged as the basis of geometry for almost 2200 years. It contains the familiar geometry of right angles, polygons, congruent triangles and parallel lines. Students taking higher mathematics at university are introduced to a range of different geometries, collectively given the designation *non-Euclidean geometry*. Such geometries are numerous and often highly unintuitive, can be difficult or impossible to visualise and only sometimes correspond to any physical reality.

NEG was first developed in Wordsworth's lifetime, simultaneously, but completely independently, by two mathematicians: the Russian Nikolai Lobachevskii and the Hungarian János Bolyai. Both of them happened also to develop the same *kind* of NEG, the kind which created the profoundest stir in mathematics and philosophy. Simply put, both mathematicians realised that, of Euclid's five fundamental postulates (or axioms) upon which his entire system was built, one was ill-defined. The so-called "parallel postulate" sought to establish as a given that two parallel lines would continue into infinity and never intersect. Under such a familiar arrangement, we can see how a straight line can be drawn and adjacent to it a point (see Appendix B, figure 1). Through that point there is only one possible line which can be drawn that will never intersect the original line, *i.e.* a single parallel line — at least in Euclidean space.

Lobachevskii (in 1929) and Bolyai (in 1923, published 1932) were each able to show that an entirely new, self-consistent geometry could be built if the parallel postulate was discarded and one assumed a space in which an infinite amount of lines could be drawn through the point, none of which would ever intersect the original line (figures 1 & 2). Such a space seems impossible, perhaps even nonsensical, but it is nonetheless mathematically consistent and deductively sound.

Bolyai was a particularly colourful intellect, something of a polymath who also wrote drama and was an excellent musician and he had no doubts about the profundity and aesthetic value of his discovery (Gray 2007, 99–100). His father, Wolfgang Bolyai, who was himself a mathematician who had grappled fruitlessly for years with the parallel postulate, warned his son of the chimera of reforming Euclid's *Elements*. A somewhat lengthy section of his letter is included here to give a sense of the monumentality of what Bolyai was attempting:

> You must not attempt this approach to parallels. I know this way to the very end. I have traversed this bottomless night, which extinguished all light and joy of my life. I entreat you, leave the science of parallels alone. I thought I would sacrifice myself for the sake of the truth. I was ready to become a martyr who would remove the flaw from geometry and return it purified to mankind. I accomplished monstrous, enormous labours; my creations are far better than those of others and yet I have not achieved complete satisfaction. For here it is true that *si paullum a summo discessit, vergit ad imum* [moving away from the mountain peak, will lead you to the abyss]. I turned back when I saw no man can reach the bottom of this night. I turned back unconsoled, pitying myself and

all mankind. (Meschowski 31, italics and translation added)

Wolfgang's final plea is as hyperbolic as the very geometry whose development it could not halt:

> For God's sake, please give it up. Fear it no less than the sensual passions because it, too, may take up all your time and deprive you of your health, peace of mind and happiness in life. (W. Bolyai in Boyer 587)

Despite the protestations of his father, Bolyai, immersed in the Romantic milieu of 1820s Vienna, continued to work away at the problem until a flash of invention suggested to him that he could simply forget the parallel postulate and develop a completely new system. He wrote back to his father in an ecstatic mood:

> The goal is not yet reached, but I have made such wonderful discoveries that I have been almost overwhelmed by them, and it would be the cause of continual regret if they were lost. When you will see them, you too will recognise it. In the meantime I can say only this: *I have created a new universe out of nothing*. All I have sent you thus far is

> but a house of cards compared to the tower. (Bonola 99; italics in Bonola)

The resonance with the passage from Book VI of *The Prelude* is immediately obvious. Bolyai developed his new geometry not by observing the natural world but by imagining possibilities, however fanciful they seemed, and by working out on paper a geometry which seemed to describe an entirely different universe.

Bolyai's work was ridiculed by some and ignored by most; meanwhile in Kazan in Russia, Lobachevkii's work did not even receive the acknowledgment of scorn and he died unsung and in poverty (Alexander 247–8). Gradually, though, in the 1850s, the work was taken up by a new vanguard of German mathematicians who recognised the crumbling Euclidean edifice and the importance of the work of Lobachevskii and Bolyai. Such names as Bernhard Riemann, Felix Klein and David Hilbert were the dominant mathematicians of the second half of the nineteenth century and they all worked to elaborate, generalise and systematise the growing number of NEGs (Gray 2007, 344–6). Riemann, in particular, was prolific in his descriptions of new geometries obeying different sets of axioms, including, most importantly, one in which no parallel lines can be drawn through a point adjacent to another line (figure 3). This NEG is the logical opposite of Bolyai's and Lobachevskii's and is best analogised with reference to the geometry of the surface of a sphere. The lines of longitude on the globe, for example, appear locally to be parallel although, globally, they all meet at the poles (if two aeroplanes take off from points adjacent to one another at the equator and set-off due North they will, for a time, travel in parallel lines, but then collide as the lines converge at the North Pole). The Bolyai–Lobachevskii NEG is normally referred to now as *hyperbolic geometry* and has no easy analogy in the real world. The geometry it describes is that of the opposite of a sphere's surface — not a readily conceivable thing (Eves 68; see figure 3)

Astonishingly, NEG turned out to describe what the universe is actually like given the distortion of spacetime which occurs around objects of great mass, such as stars and black holes and also what the universe was like in the first seconds following the Big Bang. So it was that when Einstein required an entirely new geometry to describe a relativistic universe with curved spacetime, he was afforded an "off-the-shelf" solution to this problem, by utilising NEG (Eves 69; Gray 1979, 162). Neither mathematician could possibly have anticipated the utility of their discovery and in fact the salient effect of their work, a generation later when it was actually taken up by other mathematicians, was to alter the entire conception of geometry — and thence mathematics as a whole — from being something which describes the underlying logic or mechanics of physical reality, to something which is a purely formal, purely deductive discipline capable of being explored with the imagination and without recourse to experimentation or practical application (Alexander 210, 251; Kline 1985, 217–9). Therefore, even though NEG, in a familiar turning back of history, ultimately did provide a partial account of physical reality once it was adopted by Einstein in 1916, its immediate impact was to allow for geometers to have free reign in developing alternative geometries which defied common sense; and they were prodigious in this endeavour, with the latter half of the nineteenth century yielding dozens of new systems freed from Euclidean strictures (Eves 69; Gray 2007, 190). Incidentally, only some of these correspond to any known physical phenomena, but this fact was in all senses immaterial for mathematicians of the late nineteenth century, many of whom assumed a Romantic outlook based on a few key developments in mathematics, foremost of which was NEG (Alexander 267–8). This represented perhaps the most important shift in mathematical thinking in the modern era (Alexander 210; Boyer 586; Kline 1956, 428, 431; Gray 1979, 111).

The implications of non-Euclidean geometry

Shift from Aristotelianism to platonism—Kant—relativity & Einstein—modern philosophy of maths—topology, quantum physics & string theory

It is important to note that prior to the Romantic turn of mathematics and the development of NEGs, geometry had reached its apex of utility and its strongest association with the mechanics of the universe (Alexander 177–9). Since Newton's *Principia,* the orderly, consistent, rectilinear nature of Euclidean space had been enshrined as the basis of how the objects of the physical universe are related. The Enlightenment view of geometry and mathematics more generally was of a set of rules, later more crudely, a toolkit, which allowed mathematicians, astronomers, architects, natural philosophers, engineers and others to solve problems in the real world (Alexander 49–50; Franklin; Stein 238–9). By the time of the French Revolution, French mathematicians and astronomers were convinced that the universe operated according to the laws of Newtonian mechanics, acting in Euclidean space and that all physical processes could be modelled and solved by clever application of differential equations, *i.e.* calculus (Alexander 178; Fraser 305, 307).

It was against this Enlightenment dogma that the nineteenth century mathematicians who championed NEG were pushing. There is a parallel between the rejection of Enlightenment mathematics in the mid-nineteenth century and the rejection of Enlightenment values by the seminal figures of High Romanticism (Alexander 210, 269): the late eighteenth century poets of which Wordsworth is undoubtedly the most prominent English example. It is possible to see how Wordsworth's ideas about geometry, circulated to a general audience, were in keeping with the mood of Romanticism in poetry and letters, a mood which took another generation to sweep mathematicians and a generation again for the theorems it produced to be accepted as fundamental to mathematics.

The Romantic aspect of mathematics is somewhat ignored in the humanities, where it appears to be conflated with those natural sciences that are heavily reliant on mathematics, such as physics (Baum 391–2; Alexander 174–6) . The implications of NEG for philosophy were also resisted and for decades the new characterisation of what mathematics represents was appreciated only within the mathematical community. The work of Kant was notoriously injured by NEG (Boyer 586; Kline 1956, 428–9). Indeed his famous example of synthetic *a priori* truth is Euclidean geometry; the propositions of which he held were necessarily true, regardless of experience of the world and information gathered with the senses, but which nonetheless did not have their predicate concept contained within their subject concept. He assumed our notions of space were an intrinsic part of human understanding and that Euclidean derivations about space were synthetic *a priori* knowledge, whose truth was independent of experience but needing to be deduced (Kant §§3, Book II 3.1); but this was dismantled by NEG (Posy n.66; Shapiro 88–9).

In the wake of these and other developments the chief philosophical concern of the philosophy of mathematics is the relationship between mathematical objects and reality (George & Vellman 7–11). Even still, reality can mean several things; if one means the physical universe (rather than also immaterial objects) then the relationship is unclear. Certainly there has never been observed in the physical universe anything like a perfect, Euclidean triangle with straight lines and angles which add to precisely 180 degrees (Eves 69; Hume 108). Additionally, no one has ever observed an object from Bolyai's geometry either, like a tractroid, for instance: a space with constant negative curvature where one can never approach the boundary and where there are infinite parallel lines through a single point (see Appendix B, figure 4). We have seen the geometry on a sphere (the Earth's surface) and navigators and pilots have long known that if one travels in a triangle on the surface of the Earth, the interior angles of that journey add to more than 180 degrees and that this angle sum increases with the area of the triangle. This extraordinary result demonstrates how Euclid's works were inadequate for describing how space actually works. Yet even this example is a neat abstraction and treats the surface of a sphere like a two-dimensional projection, not the three dimensional journey which actually takes place when traversing the Earth.

Furthermore, physical space on a large scale, as we have known since Einstein, actually encompasses a special kind of NEG wherein the structure of space alters locally depending on the matter contained within it (Grayling). Thus in the same way that travelling along the surface of the globe one experiences two dimensions curving through a third, the universe actually involves three dimensions warping, but without the benefit of seeing them warped *via* a higher dimension (see figure 2); it is a feat of the imagination that such spaces were conceived at all by mathematicians. Additionally, the precise kind of geometry the universe follows is not clear and it may be best described by different geometries, depending on the scale one is describing, whether a very small, quantum scale, or the macroscopic scale of the world of human experience, or the massive scale of stars, black holes and quasars (Gray 2007, 291–7). On the largest scale the answer is non-trivial. The universe as a whole could be either flat, hyperbolic or elliptic (Appendix B, figures 1–3); these three geometries, respectively, result in a fate that is either a constant rate of steady expansion where time is infinite, an accelerating expansion where time is infinite but which results in a complete dissolution and freeze of all matter and energy, or a universe with a finite end which collapses in a "big crunch" scenario.

The mere fact that space at different scales could turn out to be best described by one or another, or several geometries, but probably not all, tells us that geometries can be devised that are not bounded by or related to the physical universe (Hale & Wright 2009). This, more than anything else, suggests that geometry and perhaps all mathematics is abstract and independent of the real world; such a view is almost a given in modern schools of thought regarding the philosophy of mathematics where most mathematicians now adhere to one form or another of platonism (Folland 1121; Gray 2008, 441; Hersh 39–42); the Aristotelian view, which says mathematics is a map of the physical world, has been out of date since the time of Wordsworth's death in 1850. The term used in the literature to describe how mathematics sits "above" the physical world is that it is *conservative* over physics; in other words any physical theory which happens to be described mathematically, could also be described non-mathematically (Field 3–5). Most mathematicians now hold to the idea that mathematical objects are independent of both the physical universe and the human mind's discovery of them and that mathematics is therefore conservative over natural science (Horsten).

The evolving story of geometry is perhaps best read as a series of abstractions. Geometry — literally "earth measurement" — was first abstracted by Euclid in Greece and Brahmagupta in India, who generalised and formalised the methods used by architects, landscapers and astronomers since the earliest years of the built environment (Kline 1956, 33).These abstractions were generally applicable to any surface, any material, any project from farming, to temple building, to seafaring. Following centuries of mainly terrestrial applications, Newton then used geometry to apply these same relations to the entire universe. Following the revelations of NEG, new principles were needed which applied to the incumbent geometry of Euclid *and* the new geometries of Bolyai and others. Such principles were found in projective geometry (Kline 2007, 239; Russell 147), which looks at the properties of geometric figures that remain invariant when projected (think of the shadow of a cube being a distorted square, but with some properties still in common with the original object).

Later mathematicians found that they needed to make even higher level generalisations about spaces and shapes, but were limited by the still quantitative nature of projective geometry; in order to study space without reference, necessarily, to measurement, *topology* was developed (Grayling; Kline in Russell viii). Topology works with *qualitative* aspects of space, such that shapes or surfaces can be quantitatively deformed but still have qualitatively preserved properties (Boyer 526). Physicists working at the vanguard of quantum mechanics have since found some of these topological methods to be highly propitious for describing the possible geometry of the world on the smallest of scales (Green 162, 474; Kline 1956, 452; Kline 1985, 186). The emerging field of string theory, for example, proposes that matter is fundamentally comprised of multidimensional manifolds, through which infinitesimally small "strings" vibrate, with their precise states accounting for all known forces, including gravity, thereby explaining the seeming incompatibility of general relativity and quantum mechanics: the physics of the very large (stars, black holes) and the physics of the very small (electrons, sub-atomic particles), two adequate theories within their respective domains that, hitherto, have been irreconcilable (Greene 369–71).

It is notable that the geometry of the universe at the most fundamental level is still unknown, but that the highly abstract nature of modern mathematics means that whatever experimental evidence can be collected, it will be described by one or more of the existing geometric concepts mathematicians have accumulated; but not *all* of the NEGs, topologies, or string theoretic arrangements currently described mathematically will have any facility for describing reality. They are arguably independent worlds, created by human intelligence, or as Wordsworth would have preferred, divine intelligence. These are the soaring scientific, imaginative and philosophical implications of the untethering of geometry from physical reality and common sense notions of space. Wordsworth's rhapsodising in *The Prelude* is perhaps the only such celebration of the abstract nature of geometry to be found in the standard canon of English verse. It is certainly ahead of its time and certainly prioritises the same features of geometry as do modern mathematicians.

Chapter 3: interpretation

We have established what geometry meant to Wordsworth and how its abstract nature attracted him. We can also now appreciate the posterior move towards abstraction in NEG and in mathematics more generally. It is now possible, therefore, to situate Wordsworth's views in terms of present day opinion in the philosophy of mathematics. This chapter does so, concluding that Wordsworth's views can easily be assimilated into modern philosophical opinion and also briefly considers the manner in which we should read an apparent anticipation of later developments in mathematical thinking.

Wordsworth in the context of today's mathematics

Today's highly abstract geometry—mathematical platonism—Wordsworth as a platonist

Nowadays the philosophy of mathematics is a highly developed branch of philosophy and contains many schools of thought differentiated by technical disagreements. Contemporary arguments between say, psychological realists, *ante rem* structuralists, constructivists and myriad others are extremely recondite and here is not the place to elaborate them (Horsten). What is worth noting is that most of the perspectives in contemporary debates on the philosophy of mathematics are fundamentally to one side of the debate instigated in the late nineteenth century. At that point in time there was what would now be called an Aristotelian orthodoxy, against which platonists were pushing, bolstered by results like Bolyai's (Alexander 178; Fraser 305). The argument at that time was about the proposition that mathematics described in some essential way the operations of the external world (external to thought) as the Aristotelians would have it, versus the proposition that it described some other world or described this *and* other worlds (Detlefson; Franklin; Fraser 305–6). Alexander summarises the tension, which is a central aspect of this thesis:

> Whereas geometers from Johann Bernoulli [a late seventeenth century Swiss mathematician] to Fourier [Joseph, a late eighteenth century French mathematician] evaluated mathematics according to its utility and applicability to describing natural phenomena, the new mathematicians insisted that the field be evaluated purely in accordance with its own internal standards. And whereas Enlightenment mathematicians viewed the field as part of the natural sciences, for the new generation it was closer to the creative arts, a pure and sometimes tragic pursuit of the "inestimable treasure," truth.
>
> Simply put, whereas Enlightenment mathematics was concerned

> with this world, the new mathematics was focused on alternative universes, pure and beautiful, governed strictly by mathematical principles. [...] Like the captive in Plato's cave metaphor, who had seen the true glory of the forms, they can never again be content with life in the cave of shadows. (Alexander 181–2, quoting Gray 1979)

Note also that "Aristotelianism" is a *post facto* appellation and the inheritors of the Enlightenment view of mathematics would not have used the term themselves; in fact the label also does not even describe Aristotle's own views on mathematics particularly accurately either. Similarly, platonism is not descriptive of the views of Plato himself, but stands in to denote a view within the philosophy of mathematics which resembles some key tenets of Plato's epistemology. The platonists have since gained ascendency in the mathematical community, with the vast majority of theoretical positions and individual practitioners now aligning to a view of mathematical objects as being abstract (George & Vellman 7; Folland 1121; Gray 2008, 441; Hersh 39–42; Shabel); by abstract in this context is meant they do not have a physical or spatiotemporal reality. There are disagreements among different schools over whether these abstract objects possess existence in some immaterial sense, but that is another question well beyond this thesis. They and other contemporary schools differ from the mathematicians of the Enlightenment by accepting that there is not *necessarily* a relationship between physical reality and mathematical objects, a relationship most apparent in measurement of quantity — which is what mathematics was ultimately taken as being by most mathematicians up to the nineteenth century (Detlefson, Shabel). In the case of geometry this means today's mathematicians are in agreement with Wordsworth. So following a splitting off of views in the nineteenth century — roughly between Aristotelianism and platonism — there followed from the platonist position further splits in opinion and so on. But very few mathematicians have ever gone back to that early division and taken the path of Aristotelianism, which nowadays appears unsustainable (Boyer 661–3; Hale & Wright 2001, 156–7; Linnebo; Shapiro 202).

Linnebo writes that, in addition to abstractness, there are two other key features which define mathematical platonism and all of these features relate to the classification of mathematical objects. Platonists hold that these objects, although abstract, nonetheless possess *existence*, which we know because we utter mathematical sentences, sub-sentences of which also make sense, including to people who arrive at them independently and therefore mathematical objects exist (Linnebo). And finally, their existence is *independent* not only of physical objects (*q.v.* abstractness) but also of human thought and culture. Further, platonism is often, though not always, articulated with a theistic aspect which says that these abstract, existent objects are the product of divine creation or indeed are themselves some kind of evidence for, or manifestation of, god (Balaguer 130).

To place Wordsworth in the context of current views about the relationship between mathematical objects and physical reality might seem fraught because the of the temporal and cultural gulf, but if we take the relevant lines from the key passages we can actually infer quite a lot. Combining our knowledge of Wordsworth's views on the nature of geometry — that it is abstract, synthetic and transcendent — we can examine the key passages from *The Prelude* again and see how they are congruent to the features of modern mathematical platonism, which requires geometric objects possess abstractness, existence and independence. Recalling first the encounter with the Arab from Book V, Euclid's *Elements* is described thus:

> The one that held acquaintance with the stars,
> And wedded soul to soul in purest bond
> Of reason, undisturbed by space or time; (103–5)

The last line certainly speaks to abstractness, as clearly geometry is seen as being beyond the limits of the physical universe. The second line, however, might be read as going against the notion of independence, as we might interpret this as better fitting with the idea that mathematics is constituted by people imagining it. But returning to the passage from Book VI, it transpires that this is not the case:

> On the relation those abstractions bear
> To Nature's laws, and by what process led,
> Those immaterial agents bowed their heads
> Duly to serve the mind of earth-born man;
> From star to star, from kindred sphere to sphere,
> From system on to system without end. (123–8)

The “immaterial agents” appear to not only exist and, again, to be abstract from the physical universe, but also be preexisting and then used by humans. That they “bow their heads duly to serve the mind” of people suggests that the relation they bear to Nature’s laws is the one proffered by mathematical platonists: that the laws of nature can be described by mathematics, but that mathematics also contains a transcendent aspect which is unaffected by physical objects and independent of any theorising by intelligent beings. It seems then that Wordsworth sees geometry as being independent of human minds, but perhaps created by a divine mind:

> there, recognised

A type, for finite natures, of the one
Supreme Existence, the surpassing life
Which—to the boundaries of space and time,
Of melancholy space and doleful time,
Superior, and incapable of change,
Nor touched by welterings of passion—is,
And hath the name of, God. (VI 132–9)

Using the immutability and transcendence of mathematics to suggest a divine presence is certainly nothing new and was in fact the core tenet of both Plato and Pythagoras. More pertinently it is still adhered to by many mathematicians and theoretical physicists (Holt 172–4; Horsten). The final lines of the passage seem to confirm Wordsworth as a theistic platonist. “An independent world” is easily interpreted as meaning geometry is abstract — Wordsworth’s own word — and “Created out of pure intelligence” following from the passage above, does suggest that geometry is the product of the “Supreme Existence”; an interpretation of “pure intelligence” as meaning God, is favoured by Bruhn (57), Durrant (1970, 22, 25) and Johnson (86).

Beyond those key passages, there is further evidence in Book XI:

such sloth I could not brook,

(Too well I loved, in that my spring of life,
Pains-taking thoughts, and truth, their dear reward)
But turned to abstract science, and there sought
Work for the reasoning faculty enthroned
Where the disturbances of space and time—
Whether in matter's various properties
Inherent, or from human will and power
Derived—find no admission. (321–32)

"Abstract science" we have already noted was an amendment of the 1805 version which had "mathematics" instead. Here the poet gives us another clue as to how he perceived mathematics or the abstractions of geometry. He says they are not disturbed by "matter's various properties" or "human will and power". "[M]atter's various properties" could now align with the idea that although space is not uniform throughout the universe, because it is distorted by the matter contained within it, geometry sits above this and describes whatever configuration of space and matter happens to arise. A minority position in contemporary debates would disagree but most theoretical physicists and mathematicians hold that the arrangement of objects in the universe can be described using only a subset of extant geometries (Stewart; Resnick). The reference to human will again suggests that mathematical objects exist independently of whether humans have yet formulated them. The dominant platonist schools maintain that mathematics' coherence suggests that its relations and results are waiting to be discovered by people (Calyvan 36–7; Holt 173). Wordsworth's views of mathematics being remote from human will and power, could thus be seen as a primitive articulation of this theory, which was thoroughly unorthodox in mathematical circles at the time and which — scholars appear not to have noticed — has since become one of the standard positions held by philosophers and mathematicians.

Additionally, Wordsworth's views even obtain in modern theoretical physics. In a curious turn in intellectual history, the recrudescence of old platonic ideas in new mathematics, now extends into the physical sciences too via physics (Stewart). Nowadays when physicists try and describe space at the most fundamental level, they are forced to defer to purely mathematical descriptions, because the regions of space being studied are too small to ever see and too small even to measure with scientific instruments . No direct observations are possible and instead physicists have to infer from other observable phenomena what happens at a subatomic level and what has happened in the universe's remote past (Kline 1985, 191–4). In such cases most of the work is theoretical (purely mathematical) rather than experimental. The most prominent examples, as discussed above, are string theory and other attempts to unify quantum mechanics and general relativity. Those investigations are on the very edge of our understanding of the universe and of matter and energy. There is a sense, therefore, in which the vanguard of modern physics is completely abstract, even though it is attempting to describe the world "out there". The Calabi-Yau manifolds, for instance, which *may* be the geometry of the universe at the level of superstrings, will probably never be observable, but their geometric properties satisfy some equations and they can be described geometrically, whether they exist or not. It is not surprising that many *theoretical* physicists hold to a similar kind of platonism as do mathematicians (Holt 174–4).

Overall, Wordsworth's views on geometry, as articulated in his poetry, sit perfectly with the majority views of philosophers, mathematicians and physicists today who contemplate the relationship between geometry and the physical world. His consideration of geometry as abstract, synthetic and transcendent means he has a similar view to modern day platonists; he seems to fulfil their criteria for viewing mathematical objects (at least the ones he wrote about, those of geometry) as abstract, existent and independent. His consideration of geometry as divine places him in a smaller group, those platonists who are theistic or perhaps deistic. Although he did not say enough for us to completely delineate his views, they are simple enough that they would be uncontroversial in today's mathematical milieu; that is a striking thought when one considers how Wordsworth's reputation until quite recently, was as a somewhat flighty, anti-scientific romanticist. Instead, his writing in *The Prelude* exposes him as a poet almost unique in his admiration of mathematics and hugely valuable to a modern reader cognizant of later developments in geometry, pure mathematics and theoretical physics.

Wordsworth's prescience

Authorial intent—McGann—future research

So the obvious question of interpretation arises: how do we treat this apparent anticipation by Wordsworth of later mathematical ideas? Broadly, we have two options: we can either grant Wordsworth a degree of prescience by accepting that his transcendental philosophy admits of a view of geometry which coincides with that which was later developed by mathematicians themselves; or we can say that the author's intention is irrelevant and that the point remains that Wordsworth's platonic characterisation of geometry simply does accord with modern conceptions of geometry and therefore a modern reader can interpret the lines as resonating with modern geometers' views of their subject. (Of course there are many positions in between.) The latter view is fairly straightforwardly explicable, as a reading conducted in this vein would simply yield the insights we developed above and leave aside authorial intent; a knowledge of developments in geometry lends material to an analysis of the references to geometry in *The Prelude* and allows one to read those lines in Book VI as a celebration of what modern mathematicians would say is the purely formal nature of geometry. Such an analysis virtually ceases at that point, except to conclude that the coincidence is noteworthy.

But a reading which does take account of Wordsworth's knowledge of geometry must consider the significance of his stumbling upon, or anticipating, a perspective on geometry which was not even speculated on by most mathematicians until decades after his first draft of *The Prelude* and which was not consolidated until the late nineteenth century. Although his views could not have been informed by later developments in the field, they were demonstrably informed by his overall worldview; his early initiation to the twin cults of Euclid and Newton; and his love of abstraction, synthesis and transcendence. Further, even if we knew nothing of the author, Wordsworth, the lines of poetry in Book VI along with a knowledge of developments in the history of geometry, would together be sufficient evidence for a reading that admits that the characterisation of geometry the poem offers, fits neatly with contemporary views in the field. In fact, a *purely* formal appraisal would note the abstract nature of geometry stressed by the veritably anonymous poet and how this conjures, for a modern reader, the contemporary state of thought regarding mathematical knowledge. Perhaps the most interesting reading of the passage therefore pays comparatively less attention to the putative intentions of the author and more to the experiences, historical context and privilege of access to modern ideas of the reader.

Another theoretical path to pursue might be to engage with McGann's highly influential work on Romantic ideology, a key feature of which is the Romantic poet's idea of the timelessness of art (2). McGann argues that the poets writing in that particular epoch were subject to a particular self-defining ideology and that one element in the complex of ideas making up the ideology was the concept of the ability of poetry to convey eternal truths and for art to transcend time (135). Paradoxically, it is this notion of timelessness which made the time period unlike any previous and thus places such sentiments of timelessness firmly within a specific region of time and place (McGann 135–6). Interestingly, it is arguably in mathematics, not in poetry, that such ideas are actually realised and so poetry about the timelessness and immutability of mathematics, written in the Romantic epoch, actually does survive successive cultural transformations, and so such poetry contains a certain ironic value as yet unappreciated.

Such broader concerns are for other studies and in fact there are occasional scholarly efforts to appreciate apparently anachronistic portrayals of scientific ideas in poetry. Alberto Cappi, for example, examines Edgar Allen Poe's poem, "Eureka", finding cosmological speculations, some of which stand up well in light of modern discoveries about black holes and the age of the universe (177). Henry Grabo and Desmund King-Hele both recognise in Shelley's *Prometheus Unbound*, certain astronomical descriptions that seem to anticipate later discoveries concerning galaxies and the size of the universe (Grabo 100–10; King-Hele 578–83). And surely the metaphysical poets, who frequently drew on astronomical, cartographical and geometric conceits could be evaluated in the same manner. Perhaps there is a whole area of research ready to be undertaken that looks at past poets and their descriptions of scientific and mathematical concepts and assesses which descriptions agree with present day knowledge in those areas. The rewards for such studies are considerable, for they add a layer of meaning to the poems that was previously unappreciated and they provide for modern scientific discoveries to be honoured in verse. Such investigations will require a willingness to engage in interdisciplinary studies and perhaps even a measure of scepticism towards the very idea of disciplines and the barriers they impose.

Conclusion

Wordsworth clearly saw geometry as abstract, synthetic and transcendental. We know that he preferred Newtonian science to the nascent disciplines of chemistry and biology, apparently because of the abstractness of Newton's laws and because of their derivations using Euclidean geometry. Such ideas were unusual at the time but are not out of place in modern mathematics and therefore make him seem, in this regard at least, ahead of his time. Fittingly, it is geometry, the branch of mathematics which first undermined the Enlightenment view of the discipline as a whole (Fraser 305), which also moved Wordsworth to espouse views that were considered foolish at the time, but which are now standard within that discipline. Wordsworth's views should become known to historians of mathematics, because he celebrates geometry in a way that later generations of mathematicians would. They should also be known to Wordsworth scholars who are either unaware of his love of geometry, or who recognise it but miss the significance in terms of a whole panoply of ideas from the last 150 years of European intellectual history (Grabiner, Taylor 75–6).

This paper began by noting the affinity between lines from Wordsworth — "An independent world / Created out of pure intelligence" — and lines from Bolyai — "out of nothing I have created a strange new world." The lines have vastly different provenances and yet they refer to similar ideas.

Wordsworth's lines postulate that geometry is created by God, independent of human ingenuity or the constraints of the physical universe. Bolyai's lines express his delight in creating from his own intelligence a new set of objects. It would perhaps have been neater for this thesis if the two intelligences were the same, but a thorough reading of Wordsworth does not support this. Instead, Bolyai's discovery was one of the key elements in the development of a changing worldview within mathematics, one that would later validate the Wordsworthian position on geometry. In a sense the lines from Bolyai and those from Wordsworth were parallel; but like parallel lines in some non-Euclidean geometry, they would later meet, terminating in a point that sees the intersection of the mathematics of abstract space and the poetry of abstract ideas.

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Appendices

Appendix A: key passages from *The Prelude*, 1805 and 1850 texts

Excerpt 1: Book V, the dream of the arab and the shell

1805 ll.49–139	1850 ll.49–140
One day, when in the hearing of a friend I had given utterance to thoughts like these, He answered with a smile that in plain truth 'Twas going far to seek disquietude— But on the front of his reproof confessed That he at sundry seasons had himself Yielded to kindred hauntings, and, forthwith, Added that once upon a summer's noon While he was sitting in a rocky cave By the seaside, perusing as it chanced, The famous history of the errant knight Recorded by Cervantes, these same thoughts Came to him, and to height unusual rose While listlessly he sate, and, having closed The book, had turned his eyes towards the sea. On poetry and geometric truth (The knowledge that endures) upon these two, And their high privilege of lasting life Exempt from all internal injury, He mused—upon these chiefly—and at length, His senses yielding to the sultry air, Sleep seized him and he passed into a dream. He saw before him an Arabian waste, A desert, and he fancied that himself Was sitting there in the wide wilderness Alone upon the sands. Distress of mind Was growing in him when, behold, at once To his great joy a man was at his side, Upon a dromedary mounted high. He seemed an arab of the Bedouin tribes; A lance he bore, and underneath one arm A stone, and in the opposite hand a shell Of a surpassing brightness. Much rejoiced The dreaming man that he should have a guide To lead him through the desert; and he thought, While questioning himself what this strange freight Which the newcomer carried through the waste Could mean, the arab told him that the stone— To give it in the language of the dream— Was Euclid's Elements. 'And this', said he, 'This other', pointing to the shell, 'this book Is something of more worth.' 'And, at the word, The stranger', said my friend continuing,	One day, when from my lips a like complaint Had fallen in presence of a studious friend, He with a smile made answer, that in truth 'Twas going far to seek disquietude; But on the front of his reproof confessed That he himself had oftentimes given way To kindred hauntings. Whereupon I told, That once in the stillness of a summer's noon, While I was seated in a rocky cave By the sea-side, perusing, so it chanced, The famous history of the errant knight Recorded by Cervantes, these same thoughts Beset me, and to height unusual rose, While listlessly I sate, and, having closed The book, had turned my eyes toward the wide sea. On poetry and geometric truth, And their high privilege of lasting life, From all internal injury exempt, I mused, upon these chiefly: and at length, My senses yielding to the sultry air, Sleep seized me, and I passed into a dream. I saw before me stretched a boundless plain Of sandy wilderness, all black and void, And as I looked around, distress and fear Came creeping over me, when at my side, Close at my side, an uncouth shape appeared Upon a dromedary, mounted high. He seemed an Arab of the Bedouin tribes: A lance he bore, and underneath one arm A stone, and in the opposite hand a shell Of a surpassing brightness. At the sight Much I rejoiced, not doubting but a guide Was present, one who with unerring skill Would through the desert lead me; and while yet I looked and looked, self-questioned what this freight Which the new-comer carried through the waste Could mean, the Arab told me that the stone (To give it in the language of the dream) Was "Euclid's Elements;" and "This," said he, "Is something of more worth;" and at the word

‘Stretched forth the shell towards me, with command
That I should hold it to my ear. I did so
And heard that instant in an unknown tongue,
Which yet I understood, articulate sounds,
A loud prophetic blast of harmony,
And ode in passion uttered, which foretold
Destruction to the children of the earth
By deluge now at hand. No sooner ceased
The song, but with calm look the arab said
That all was true, that it was even so
As had been spoken, and that he himself
Was going then to bury those two books
The one that held acquaintance with the stars,
And wedded man to man by purest bond
Of nature, undisturbed by space or time;
Th’ other that was a god, yea many gods,
Had voices more than all the winds, and was
A joy, a consolation, and a hope.’
My friend continued, ‘Strange as it may seem
I wondered not, although I plainly saw
The one to be a stone, th’ other a shell,
Nor doubted once but that they both were books,
Having a perfect faith in all that passed.
A wish was now engendered in my fear
To cleave unto this man, and I begged leave
To share his errand with him. On he passed
Not heeding me; I followed, and took note
That he looked often backward with wild look,
Grasping his twofold treasure to his side.
Upon a dromedary, lance in rest,
He rode, I keeping pace with him; and now
I fancied that he was the very knight
Whose tale Cervantes tells, yet not the knight,
But was an arab of the desert too,
Of these was neither, and was both at once.
His countenance meanwhile grew more disturbed,
And looking backwards when he looked I saw
A glittering light, and asked him whence it came.
”It is”, said he, ”The waters of the deep
Gathering upon us.” Quickening then his pace
He left me; I called after him aloud;
He heeded not, but with his twofold charge
Beneath his arm—before me full in view—
I saw him riding o’er the desart sands
With the fleet waters of the drowning world
In chace of him; whereat I waked in terror,
And saw the sea before me, and the book
In which I had been reading at my side.’

Stretched forth the shell, so beautiful in shape,
In colour so resplendent, with command
That I should hold it to my ear. I did so,
And heard that instant in an unknown tongue,
Which yet I understood, articulate sounds,
A loud prophetic blast of harmony;
An Ode, in passion uttered, which foretold
Destruction to the children of the earth
By deluge, now at hand. No sooner ceased
The song, than the Arab with calm look declared
That all would come to pass of which the voice
Had given forewarning, and that he himself
Was going then to bury those two books:
The one that held acquaintance with the stars,
And wedded soul to soul in purest bond
Of reason, undisturbed by space or time;
The other that was a god, yea many gods,
Had voices more than all the winds, with power
To exhilarate the spirit, and to soothe,
Through every clime, the heart of human kind.
While this was uttering, strange as it may seem,
I wondered not, although I plainly saw
The one to be a stone, the other a shell;
Nor doubted once but that they both were books,
Having a perfect faith in all that passed.
Far stronger, now, grew the desire I felt
To cleave unto this man; but when I prayed
To share his enterprise, he hurried on
Reckless of me: I followed, not unseen,
For oftentimes he cast a backward look,
Grasping his twofold treasure.—Lance in rest,
He rode, I keeping pace with him; and now
He, to my fancy, had become the knight
Whose tale Cervantes tells; yet not the knight,
But was an Arab of the desert too;
Of these was neither, and was both at once.
His countenance, meanwhile, grew more disturbed;
And, looking backwards when he looked, mine eyes
Saw, over half the wilderness diffused,
A bed of glittering light: I asked the cause:
"It is," said he, "the waters of the deep
Gathering upon us;" quickening then the pace
Of the unwieldy creature he bestrode,
He left me: I called after him aloud;
He heeded not; but, with his twofold charge
Still in his grasp, before me, full in view,
Went hurrying o'er the illimitable waste,
With the fleet waters of a drowning world
In chase of him; whereat I waked in terror,
And saw the sea before me, and the book,
In which I had been reading, at my side.

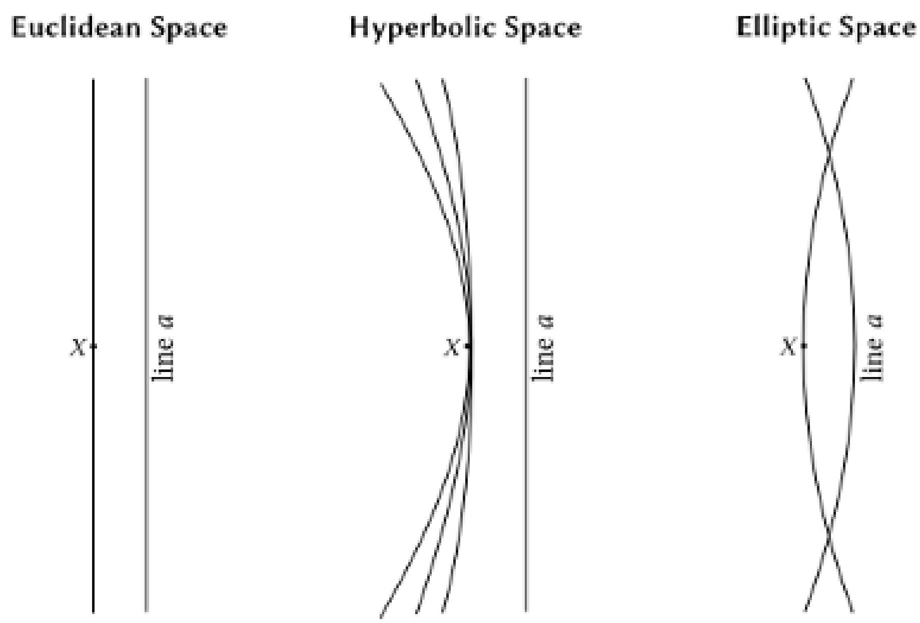
Excerpt 2: Book VI, Cambridge and the Alps

1805 (ll.135–187)	1850 (ll.115–167)
Yet must I not entirely overlook	Yet may we not entirely overlook
The pleasure gathered from the elements	The pleasure gathered from the rudiments
Of geometric science. I had stepped	Of geometric science. Though advanced
In these inquiries but a little way,	In these inquiries, with regret I speak,
No farther than the threshold—with regret	No farther than the threshold, there I found
Sincere I mention this—but there I found	Both elevation and composed delight:
Enough to exalt, to chear me and compose.	With Indian awe and wonder, ignorance
With Indian awe and wonder, ignorance	pleased
Which even was cherished, did I meditate	With its own struggles, did I meditate
Upon the alliance of those simple, pure	On the relation those abstractions bear
Proportions and relations, with the frame	To Nature's laws, and by what process led,
And laws of Nature—how they could become	Those immaterial agents bowed their heads
Herein a leader to the human mind—	Duly to serve the mind of earth-born man;
And made endeavours frequent to detect	From star to star, from kindred sphere to sphere,
The process by dark guesses of my own.	From system on to system without end.
Yet from this source more frequently I drew	
A pleasure calm and deeper, a still sense	More frequently from the same source I drew
Of permanent and universal sway	A pleasure quiet and profound, a sense
And paramount endowment in the mind,	Of permanent and universal sway,
An image not unworthy of the one	And paramount belief; there, recognised
Surpassing life, which—out of space and time,	A type, for finite natures, of the one
Nor touched by welterings of passion—is,	Supreme Existence, the surpassing life
And hath the name of, God. Transcendent peace	Which—to the boundaries of space and time,
And silence did await upon these thoughts	Of melancholy space and doleful time,
That were a frequent comfort to my youth.	Superior, and incapable of change,
	Nor touched by welterings of passion—is,
And as I have read of one by shipwreck thrown	And hath the name of, God. Transcendent peace
With fellow sufferers whom the waves had	And silence did await upon these thoughts
spared	That were a frequent comfort to my youth.
Upon a region uninhabited,	
An island of the deep, who having brought	'Tis told by one whom stormy waters threw,
To land a single volume and no more—	With fellow-sufferers by the shipwreck spared,
A treatise of geometry—was used,	Upon a desert coast, that having brought
Although of food and clothing destitute,	To land a single volume, saved by chance,
And beyond common wretchedness depressed,	A treatise of Geometry, he wont,
To part from company and take this book,	Although of food and clothing destitute,
Then first a self-taught pupil in those truths,	And beyond common wretchedness depressed,
To spots remote and corners of the isle	To part from company and take this book
By the seaside, and draw his diagrams	(Then first a self-taught pupil in its truths)
With a long stick upon the sand, and thus	To spots remote, and draw his diagrams
Did oft beguile his sorrow, and almost	With a long staff upon the sand, and thus
Forget his feeling: even so—if things	Did oft beguile his sorrow, and almost
Producing like effect from outward cause	Forget his feeling: so (if like effect
So different may rightly be compared—	From the same cause produced, 'mid outward

So was it with me then, and so will be With poets ever. Mighty is the charm Of those abstractions to a mind beset With images, and haunted by itself, And specially delightful unto me Was that clear synthesis built up aloft So gracefully, even then when it appeared No more than as a plaything, or a toy Embodied to the sense—not what it is In verity, an independent world Created out of pure intelligence.	things So different, may rightly be compared), So was it then with me, and so will be With Poets ever. Mighty is the charm Of those abstractions to a mind beset With images, and haunted by herself, And specially delightful unto me Was that clear synthesis built up aloft So gracefully; even then when it appeared Not more than a mere plaything, or a toy To sense embodied: not the thing it is In verity, an independent world, Created out of pure intelligence.

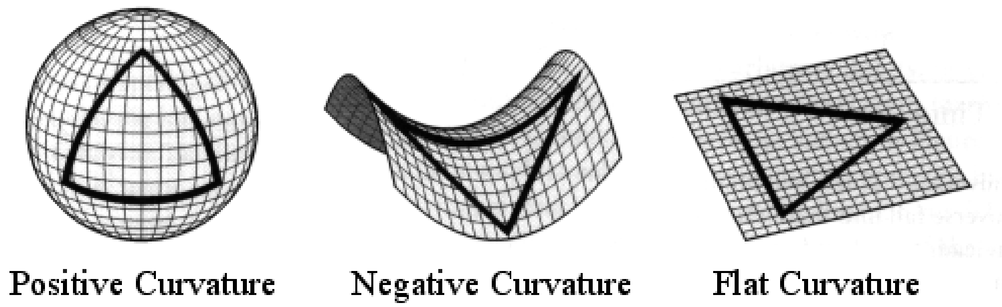
Appendix B: diagrams illustrating non-Euclidean geometries

Figure 1: parallel lines in euclidean, hyperbolic and elliptical geometry



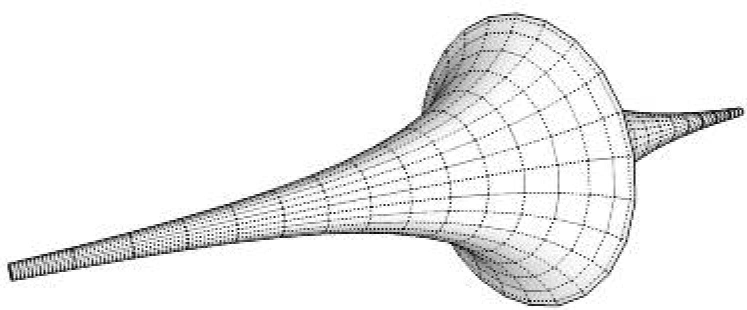
In euclidean space, given a straight line *a* and a point *x* adjacent to it, there is only one possible line that can be drawn through *x* that will not intersect with *a*: a parallel line. But in Bolyai and Lobachevskii's geometry (hyperbolic) there are an infinite spray of lines that could be drawn through *x*, all of which are locally parallel, but which fan out and move away from the line *a*. In one of Riemann's geometries (elliptical) it is impossible to draw a line through *x* that will not intersect *a* at some point; this is similar to lines of longitude on a globe. (Image used under a Creative Commons licence, authored by Pedro Rosario.)

Figure 2: space with different curvatures



The explanation for how different rules for parallel lines can obtain in different geometries is that they assume different curvatures of space. Elliptical geometry describes the geometry of a space of constant positive curvature (like the surface of a sphere). Hyperbolic geometry describes a space of constant negative curvature (see Figure 3, the pseudosphere). Euclidean geometry describes a space with flat curvature. (Image from the Department of Astronomy of the University of Florida's website.)

Figure 3: a tractroid or pseudosphere



The tractroid is the opposite of a sphere, with the middle "lip" having a radius and its overall volume being equal to a sphere of the same radius. But although its volume is finite its extent is infinite as the "tails" continue forever in each direction. (Image courtesy of Google Sketchup.)